Rethinking Calculus and Modeling

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GOALS:

• to rethink the content of traditional calculus courses, so their relevance to the sciences would be enhanced, in realistic and practical ways.

• to help the science students better understand and appreciate, and begin to utilize, the important role that mathematical modeling can play in scientific investigation.

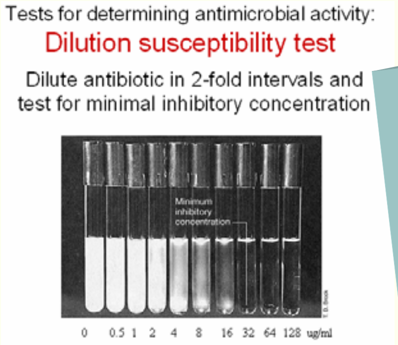
• to teach math-inclined science students how to construct and analyze mathematical models of scientific processes.

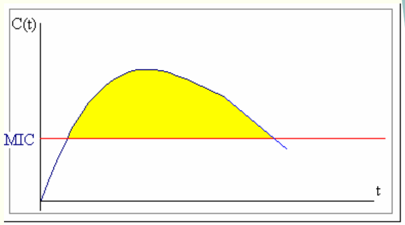
Important mathematical topics that have been absent or underrepresented in standard calculus courses:

- multivariate calculus (Currently, most science students see quantities that depend on two or more independent variables much earlier in their science courses than they do in mathematics.)
- worst-case error estimates and practical estimation
- responsible data set management, including regression techniques
- discrete probability
- linear algebra, as it relates to dynamical systems
- models
- modern and relevant examples and applications

To make room for these new topics, we did two things:

Omitted some less-relevant (to the sciences and to modern applied mathematicians) math topics, such as endpoint tests for Taylor series, and the traditional physics/geometry applications of integrals.
Opened the course only to those students who already have a good calculus background (typically, a good calculus course in high school).





C(t) = the drug concentration at time t
Yellow area is the number of MIC-hours

In order for an antibiotic to kill a bacteria colony, the concentration of antibiotic at the infection site must be high enough. The lowest effective concentration is called the minimum inhibitory concentration (MIC). The MIC depends on the choice of antibiotic, as well as the particular type of bacteria. In addition, this effective concentration must be maintained for a sufficiently long period of time, so that the entire colony is eliminated.

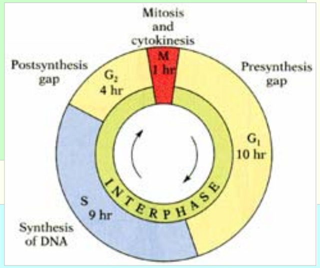
The total effective exposure (incorporating both the concentration and the amount of time) is measured in MIC-hours. One MIC-hour is a one-hour period with the drug concentration one unit (micrograms/ml) above the MIC. Overall treatment is determined by the total # of MIC-hours. (e.g. If MIC= 8mg/ml and the drug concentration is 10 mg/ml for 3 hours, then a total of 6 MIC-hours are accumulated.

Math 231-232: Scientific Calculus I and II:

The MIC-hour concept is investigated at two different points during the course sequence: as an application of area between curves in the first semester, where the concentration c(t) is given by formula without much justification or as a data table, and again in the second semester in the differential equation unit, where the students construct simple pharmacokinetics models, whose solutions are the concentration functions c(t).

In the modeling course, the strategy is to teach the students some modeling principles, then study models in various areas of biology and medicine. The topics and their sequencing were carefully planned to introduce successively higher-level model-building situations and analysis skills. The bio-medical topics for the course are, in sequence:

- Biological control of pest populations
- Tumor growth dynamics
- Pharmacokinetics
- Models of chemotherapy
- Epidemiology
- Interacting populations
- Leukemia dynamics
- Immunology of the HIV virus
- Enzyme kinetics



Biomedical Modeling Course:
Cancer Spotlights

Three Spotlights (one class period each) during the semester:
Spotlight #1: Models of Tumor Growth
Spotlight #2: Models of Chemotherapy
Spotlight #3: Models of Leukemia Dynamics
Notes:
These spotlights are strategically placed during the semester so that the students have the skills to understand the models.
Spotlight #2 combines tumor growth models from Spotlight #1 with pharmacokinetics models, which is the unit just completed
The goal of Spotlight #3 is to compare early-, uniform-, and late-intensity treatment regimens.

In order for an antibiotic to kill a bacteria colony, the concentration of antibiotic at the infection site must be high enough. The lowest effective concentration is called the minimum inhibitory concentration (MIC). The MIC depends on the choice of antibiotic, as well as the particular type of bacteria. (it can even vary between different strains of the same bacteria species. In addition, this effective concentration must be maintained for a sufficiently long period of time, so that the entire colony is eliminated.

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