The Introductory Life Science Mathematics Sequence

EMORY COLLEGE

Mathematics & Computer Science in collaboration with the Center for Science Education and the Department of Biology

Objectives of the Courses

Mathematical, computational and statistical methods are of ever-growing importance in the life sciences. We introduce the basic quantitative tools required in modern life science research, with attention to the recommendations of BIO 2010 [NRC] and Math & Bio 2010 [MAA]. Courses emphasize

- modeling change in biological systems via discrete dynamics, continuous differential equations, and stochastic processes, and
- organizing and analyzing data in information-intensive application areas such as molecular evolution and genetics.

Researchers in the biological sciences and health science professionals must be discerning readers of the research literature. In particular, critical evaluation of the construction of, and conclusions drawn from, statistical studies is indispensable. This requires

- an understanding of probability theory, as the underpinning of inferential statistics, and
- exposure to a variety of statistical methods used to establish confidence and test hypotheses

Challenges

- Balance diverse mathematical backgrounds of the students.
- Integrate motivating life science phenomena with mathematical methods.
- Coordinate several texts and web materials as course source materials.
- Assess curriculum, teaching methods, supplemental instructions and student learning.
- Attune courses with major programs in Biology and other life science majors.

References

1. *BIO 2010 – Transforming Undergraduate Education for Future Research Biologists*, National Research Council, The National Academies Press [Washington DC, 2003] – specifies mathematical, statistical and computational needs, see Appendix F.
2. *Calculus Lite* (3rd), F. Morgan, AK Peters [Wellesley, MA, 2001] – provides the differential and integral calculus materials for Math 115 – Life Science Mathematics 1.
3. *CCP Materials*, L. Moore and D. Smith, www.math.duke.edu/education/modules2/materials – supports DE modeling topics.
4. *Chance in Biology – Using Probability to Explore Nature*, M. Denny and S. Gaines, Princeton University Press [Princeton, 2000] – provides introductory material on probability and explores the stochastic nature of nature.
5. *Math & Bio 2010*, L. Steen [ed], Mathematical Association of America [2005] – essays on the interrelations among biology, computing and mathematics for undergraduates.
6. *Mathematical Models in Biology*, E. Allman and J. Rhodes, Cambridge University Press [Cambridge 2004] – the main source of life science topics throughout the sequence.

Life Science Mathematics I

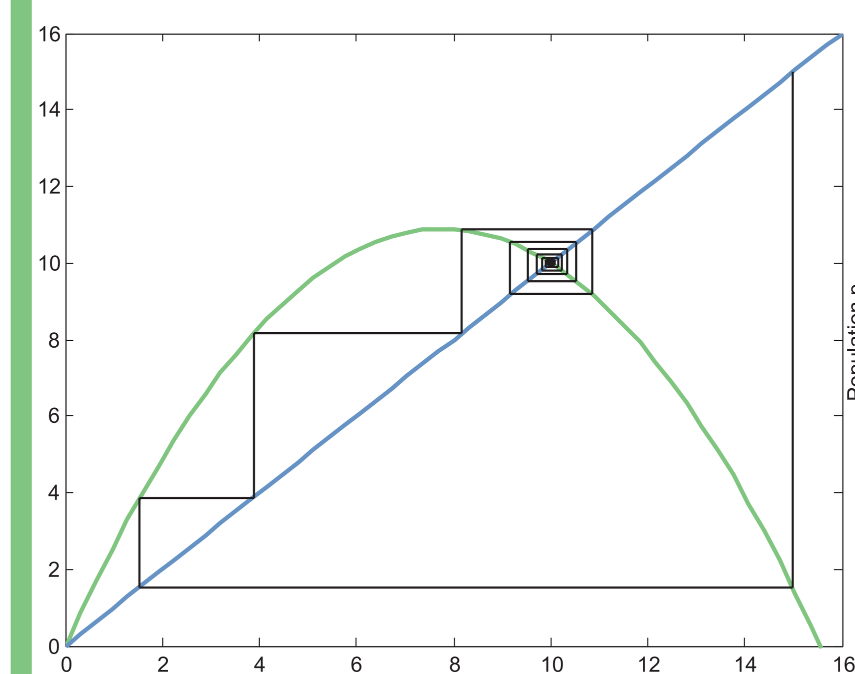
Course Outline

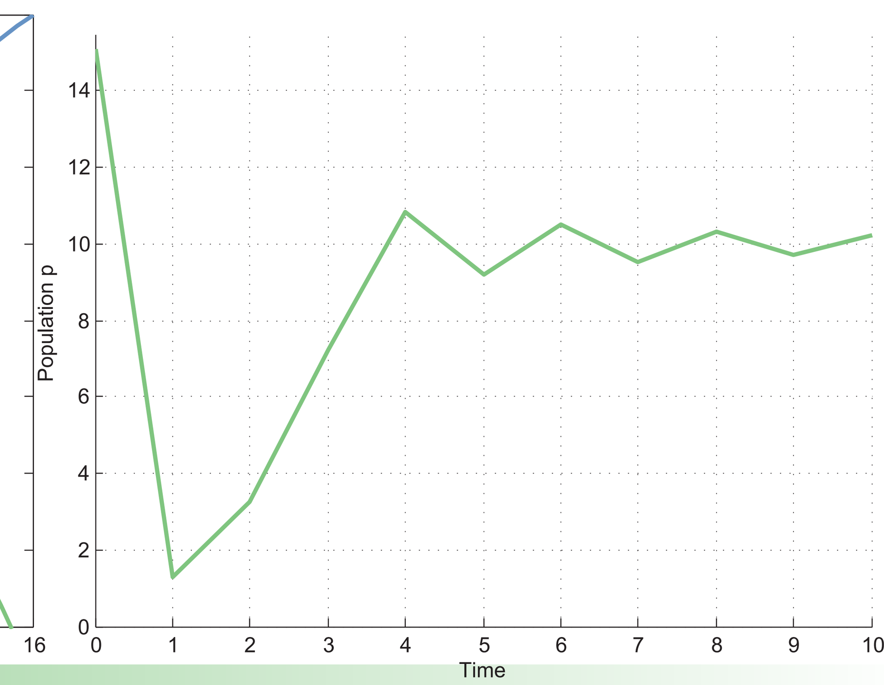
- Motivate methods through difference equation models of population dynamics
- Introduce differential calculus to describe continuous change.
- Explore discrete & continuous models of populations, infectious disease and predator-prey systems.
- Include analysis via cobwebbing, phase lines, and characterization of equilibrium stability via derivatives.
- Introduce antidifferentiation and pure-time differential equations.
- Derive autonomous differential equations from modeling processes already seen in population growth, epidemics and predator prey systems.
- Develop the definite integral, linked to antidifferentiation by the Fundamental Theorem of Calculus, and include applications to area and averaging.

Course Examples

Logistic Growth: Non-linear Difference Equations for Population Growth

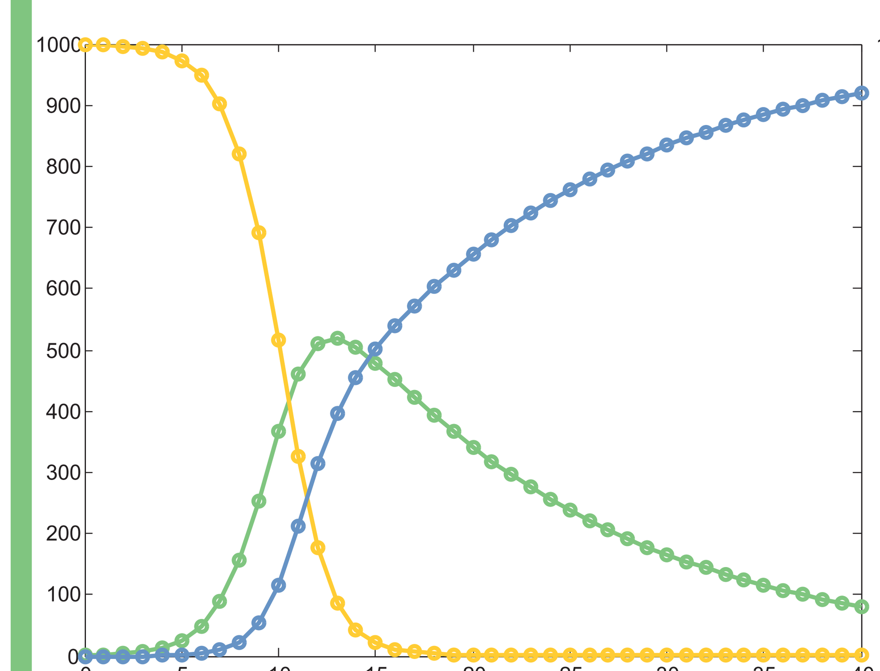
Cobwebbed updating function and corresponding solution for logistic population model, $p_{t+1} = p_t + 1.8p_t(1-p_t/10)$.
Does the initial condition determine the long term behavior of the population?

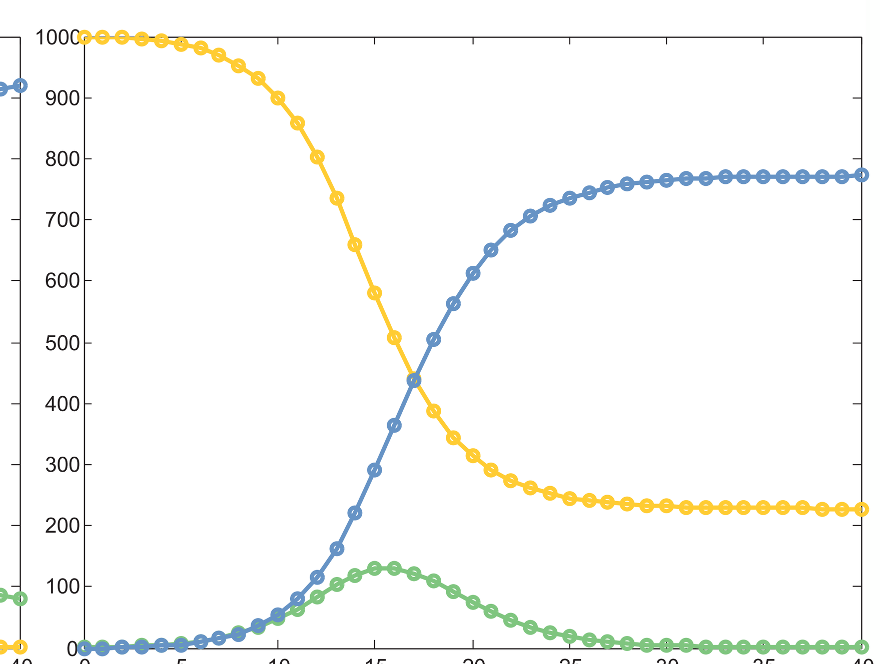




S-I-R Simulations: Infectious Disease Modeling

Examples of *S-I-R* simulations for: $S_{t+1} = S_t - \alpha S_t I_t$, $I_{t+1} = I_t + \alpha S_t I_t - \gamma I_t$. In first image, α=.001,γ=.07; in second α=.001,γ=.5; susceptibles are yellow, infectives are green, removeds are blue. What explains the difference in the outcomes? Does the model predict observable behavior?





Life Science Mathematics II

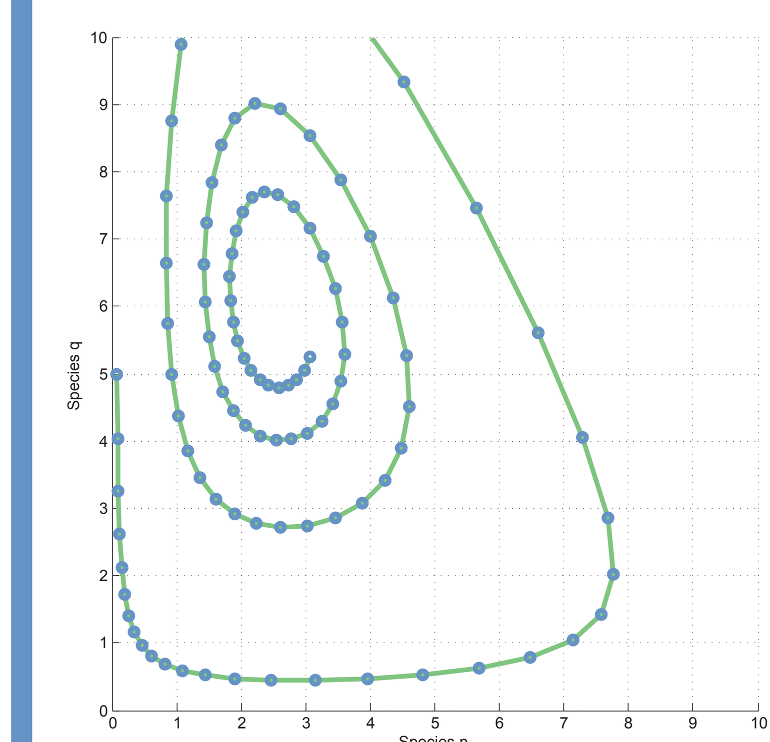
Course Outline

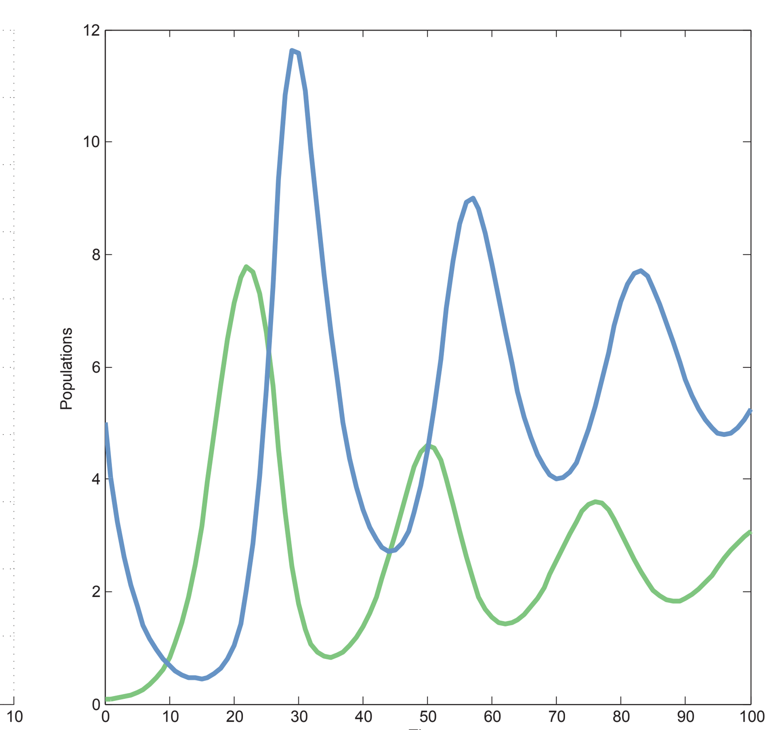
- Examine systems of differential equations describing predator-prey systems and the SIR disease model.
- Describe qualitative behavior with slope fields and phase plane diagrams.
- Use genetics and molecular evolution to motivate probability and statistics.
- Probability theory includes: discrete spaces, conditional probability, binomial and geometric distributions, random variables, expectation; probability density functions, the normal distribution, and the Central Limit Theorem.
- Statistical methods include: descriptive and inferential statistics, with emphasis on hypothesis testing, sampling distributions and analysis of variance.
- Include probabilistic topics such as random walks applied to diffusion and genetic drift, and statistical tools such as the χ^2-test, contingency tables, goodness-of-fit, and nonparametric tests.

Course Examples

Predator-Prey: Systems of Differential Equations for Interacting Populations

Phase plane trajectory and solution for $dp/dt=0.4p(1-p/10)-0.05pq$, $dq/dt = 0.2q + 0.08pq$. Is this a predator-prey system? Which species is the prey?





Hypothesis Testing: Approximation of Sampling Distributions with Normal Curve

The left histogram is the frequency distribution of 1000 means of samples of size $n=3$ from an exponential distribution along with its CLT-predicted normal approx. On the right, $n=30$, where the normal approx is a better fit. Can you approximate the variances? Can you account for the difference in the variances?